In triangles APB find $AP_1=166.77$, $AP_2=572.10$, $AP_3=57.96$, $AP_4=600.81$.

Also solved by J. Scheffer and H. C. Feemster.

CALCULUS.

320. Proposed by J. F. LAWRENCE, Stillwater, Oklahoma.

Show that, if $u=1+A_1x+\frac{1}{2!}A_2x^2+\frac{1}{3!}A_3x^3+\dots$ where the quantities A are connected by the relation $A_m=mA_{m-1}-\frac{1}{2}(m-1)(m-2)A_{m-3}$, then $\log[u(1-x)^{\frac{1}{2}}]=\frac{1}{2}x+\frac{1}{4}x^2$. [From Forsyth's *Differential Equations*, p. 48.]

I. Solution by E. B. ESCOTT, Ann Arbor, Michigan.

$$u=1+A_1x+\frac{1}{2!}A_2x^2+\dots+\frac{A_m}{m!}x^m+\dots$$

where $A_m=mA_{m-1}-\frac{1}{2}(m-1)(m-2)A_{m-3}$.

Form the expression which has for the coefficient of x^m,

$$A_m-mA_{m-1}+\tfrac{1}{2}(m-1)(m-2)A_{m-3}$$

multiplied by some factor. Then such an expression will have only a finite number of terms. Such an expression is

$$u-xu+\tfrac{1}{2}\int x^2u\,dx;$$

also its derivative

$$(1-x)\frac{du}{dx}+(\tfrac{1}{2}x^2-1)u.$$

Therefore, we have for the differential equation of u,

$$(1-x)\frac{du}{dx}+(\tfrac{1}{2}x^2-1)u=0.$$

Separating variables,

$$\frac{du}{u}+\frac{\frac{1}{2}x^2-1}{1-x}dx=0.$$

whence

$$\log u(1-x)^{\frac{1}{2}}=\frac{x}{2}+\frac{x^2}{4},$$

the constant of integration being zero, since when $x=0$, $u=1$, and $\log 1=0$.

II. Solution by A. M. HARDING, University of Arkansas, Fayetteville, Arkansas.

From the given relation we have

$$A_1=1,\ A_2=2,\ \dots,\ \tfrac{1}{2}\frac{A_{m-3}}{(m-3)!}=\frac{mA_{m-1}}{(m-1)!}-\frac{mA_m}{m!}\dots(1).$$

Let $\phi(x)=(1-\frac{x^2}{2})(1+A_1x+\frac{A_2}{2!}x^2+\dots+\frac{1}{m!}x^m+\dots)(1+x+x^2+\dots+x^m+\dots)$

$$=[1+A_1x+(\frac{A_2}{2!}-1)x^2+\dots+(\frac{A_m}{m!}-\tfrac{1}{2}\frac{A_{m-2}}{(m-2)!})x^m+\dots]\times$$

$$[1+x+x^2+\dots+x^m+\dots]$$

Let us assume that both these series are absolutely convergent. Multiply them in the ordinary way and obtain a series $\Sigma c_m x^m$ where c_m, after reduction, has the value

$$c_m=\frac{A_m}{m!}+\frac{A_{m-1}}{(m-1)!}+\tfrac{1}{2}\left[\frac{A_{m-2}}{(m-2)!}+\frac{A_{m-3}}{(m-3)!}+\dots+\frac{A_2}{2!}+A_1+1\right].$$

From the relation (1) we have

$$\left\{\begin{array}{l}
\tfrac{1}{2}=\frac{3A_2}{2!}-\frac{3A_3}{3!}=1+A_1+\frac{A_2}{2!}-\frac{3A_3}{3!} \quad (\text{since } \frac{3A_2}{2!}=1+A_1+\frac{A_2}{2!}=3. \\[2ex]
\tfrac{1}{2}A_1=\frac{4A_3}{3!}-\frac{4A_4}{4!}. \\[2ex]
\tfrac{1}{2}\frac{A_2}{2!}=\frac{5A_4}{4!}-\frac{5A_5}{5!}. \\[1ex]
\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\[1ex]
\tfrac{1}{2}\frac{A_{m-2}}{(m-2)!}=(m+1)\frac{A_m}{m!}-(m+1)\frac{A_{m+1}}{(m+1)!}.
\end{array}\right.$$

Adding these equations, we obtain

$$0=c_m-(m+1)\frac{A_{m+1}}{(m+1)!}.$$

$\therefore c_m=(m+1)\dfrac{A_{m+1}}{(m+1)!}$, and $\phi(x)=\Sigma c_m x^m=A_1+A_2x+\dfrac{1}{2!}A_3x^2+\ldots$
$+\dfrac{1}{m!}A_{m+1}x^m$.

$\therefore \phi(x)=\dfrac{du}{u}$. But $\phi(x)\equiv(1-\dfrac{x^2}{2})(1-x)^{-1}.u=\dfrac{u}{2}[1+x+\dfrac{1}{1-x}]$.

$\therefore \dfrac{du}{u}-\dfrac{dx}{2(1-x)}=(\frac{1}{2}+\dfrac{x}{2})dx$. Integrating, we obtain

$$\log u+\tfrac{1}{2}\log(1-x)=\frac{x}{2}+\frac{x^2}{4}+c.$$

Now when $x=0$, $u=1$, and hence $c=0$.

$$\therefore \log[u(1-x)^{\frac{1}{2}}]=\frac{x}{2}+\frac{x^2}{4}.$$

C. N. Schmall should have received credit for solving 317.

MECHANICS.

---

357. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

A portion of a circular cylinder cut off by two planes through the axis rests with its curved surface on two rough horizontal rails parallel to its axis, the coefficients of friction μ_1, μ_2 at upper and lower rails respectively. If the body is in limiting equilibrium at both rails when the plane through the axis and the center of gravity is perpendicular to both rails, find the distance of the center of gravity in terms of the distance betwen the rails, the inclination of their plane to the horizon, and the cofficients of friction.

358. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Two heavy particles connected by a string, length l, lie one on each of two inclined planes with common horizontal edge and of angles α and β. The inclination of the string to the edge varies as the inclination to the horizon of a simple pendulum of length $l(\sin\alpha+\sin\beta)$.

No solutions of these problems have been received.

259. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A uniform beam of the weight W, rests on a horizontal plane, and leans against a vertical wall, but so as *not* to lie in a vertical plane. Denoting the pressure upon the horizontal and vertical planes, respectively, by x